The Decoherence of Schrödinger's State: A Quantitative Analysis of Fidelity in Noisy Quantum Simulations

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Abstract. *The viability of algorithms on Noisy Intermediate-Scale Quantum (NISQ) computers is limited by operational errors and decoherence. To optimize execution, it is crucial to understand how circuit complexity (depth and number of gates), statistical sampling (shots), and result fidelity interact. This work investigates that relationship through large-scale simulations of 100,000 random circuits under a physically motivated noise model with temporal decoherence (T1/T2) and gate errors. The analysis shows a strong negative correlation between circuit complexity and Hellinger fidelity, revealing a "fidelity ceiling" where increasing shots reduces statistical error but cannot overcome noise. For complex circuits, the number of gates is identified as a stronger predictor of error than circuit depth.*
Keywords:*Quantum Computing; Hellinger Fidelity; Decoherence; Statistical Sampling*

1. Introduction

Quantum computing emerges as a promising field for solving complex problems, but its practical viability depends on the fidelity of the results, which is limited by hardware noise and statistical sampling [1]. Noise, arising from decoherence and gate errors, degrades quantum information, while the number of *shots* determines the precision in reconstructing the final state.

In this work, we quantitatively analyze the relationship between circuit complexity, the number of *shots*, and final fidelity, with the goal of determining how many shots are required to obtain accurate results in noisy quantum circuits. To this end, we simulated 100,000 random circuits in Qiskit Aer using a physically motivated noise model and evaluated Hellinger fidelity. The results show that circuit complexity, particularly the number of gates and qubits, is the main predictor of fidelity loss. Additionally, we identify a "fidelity ceiling," where increasing the number of *shots* reduces statistical uncertainty but does not reverse the degradation caused by noise.

2. Theoretical Background

The evolution of a closed quantum system is governed by the Schrödinger equation, and its state is described by a vector $|\psi\rangle$. According to the Born rule, measurement is probabilistic, and a single *shot* provides one sample of the final state distribution. To reconstruct the full distribution, multiple *shots* are required, defining the precision of the observation [2].

NISQ devices interact with their environment, experiencing decoherence and gate errors. The main mechanisms are energy relaxation (T1) and phase damping (T2) [3]. The noise model used in this work simulates these combined effects based on the physical parameters of the hardware.

To quantify the impact of noise and sampling, we use Hellinger fidelity, which measures the overlap between the experimental distribution (P) and the theoretical one (Q):

$$F_H(P,Q) = \Big(\sum_i \sqrt{p_i q_i}\Big)^2$$

Ranging from 0 (completely different) to 1 (identical), this metric is robust for assessing the quality of the result [4].

3. Methodology

To investigate the relationship between circuit complexity, statistical sampling, and fidelity, we conducted a large-scale computational experiment divided into three steps: generation of random quantum circuits, simulation in ideal and noisy environments, and data analysis.

We generated 100,000 circuits with number of qubits $W \in [5, 12]$ and depth $D \in [30, 50]$, applying 1-qubit gates ($\{H, X, Y, Z, S, T\}$) and 2-qubit gates ($\{CX, CZ, Swap\}$) until reaching the target depth. This procedure ensures a dataset diverse in structural complexity.

3.1. Simulation and Noise Model

Simulations were performed using Qiskit Aer:

Ideal Simulation: AerSimulator without noise, with 16,384 shots, providing the theoretical distribution Q.

Noisy Simulation: AerSimulator with a physically motivated noise model simulating NISQ devices. The parameters were chosen as representative values of contemporary superconducting hardware to ensure the generality of the results. It included: - *Decoherence*: thermal relaxation errors (T1 = 80μs, T2 = 60μs), applied according to realistic gate durations (50ns for 1-qubit vs. 400ns for 2-qubit) that reflect the higher complexity of entangling operations. - *Gate Errors*: depolarizing errors with probabilities (0.2% for 1-qubit, 2% for 2-qubit) selected to model the significant fidelity gap between single- and two-qubit gates in current devices.

Each noisy circuit was executed with different numbers of shots (128, 512, 1024, 4096, and 8192). The Hellinger fidelity $F_H(P, Q)$ was computed to quantify the similarity between the noisy and ideal results. For each circuit, we recorded: index, qubits, actual depth, number of gates, shots, execution time, and resulting fidelity.

4. Results and Discussion

The simulation of 100,000 noisy circuits provided a robust dataset. The average final fidelity (8192 shots) was 0.949 ± 0.023, ranging from 0.741 to 0.999, indicating a strong dependence on the complexity of each circuit.

Figure 1 shows the convergence of Hellinger fidelity with increasing shots for three representative cases: the 10 best circuits (highest fidelities), the 10 worst circuits (lowest fidelities), and 10 circuits around the median. Statistical sampling mitigates measurement uncertainty, but in all cases the results converge to a *fidelity ceiling* imposed by hardware noise, which cannot be overcome by increasing the number of shots alone. Analysis of structural predictors shows a strong negative correlation between fidelity and

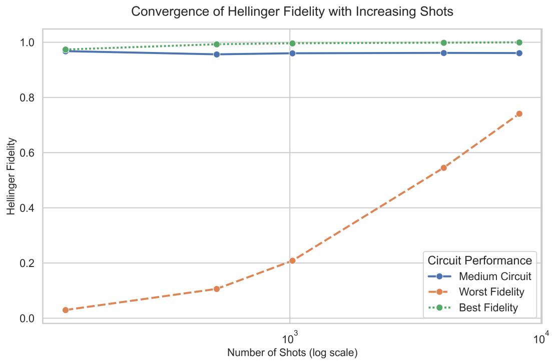

Figure 1. Hellinger fidelity convergence with increasing shots for three representative circuits.

the number of gates (Figure 2), while depth shows a weak correlation (Figure 3), suggesting that the total number of operations is the main factor driving fidelity degradation.

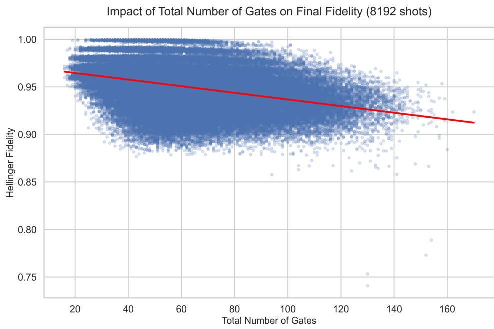

Figure 2. Relation between total number of gates and final Hellinger fidelity.

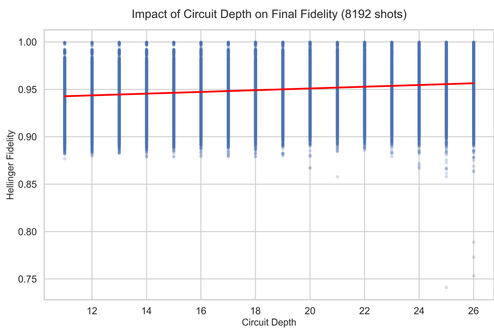

Figure 3. Relation between circuit depth and final Hellinger fidelity.

To synthesize the analysis of complexity, the heatmap in Figure 4 summarizes the average fidelity across the entire parameter space. The color gradient visually confirms that fidelity systematically degrades with both an increasing number of qubits (y-axis) and circuit depth (x-axis). The plot serves as a performance map for our simulated NISQ device, clearly delineating the ”safe” regions of high fidelity (e.g., few qubits, shallow depth, approx. 0.98) from the regions of higher vulnerability where reliable computation becomes challenging (e.g., 12 qubits, deep circuits, approx. 0.914).

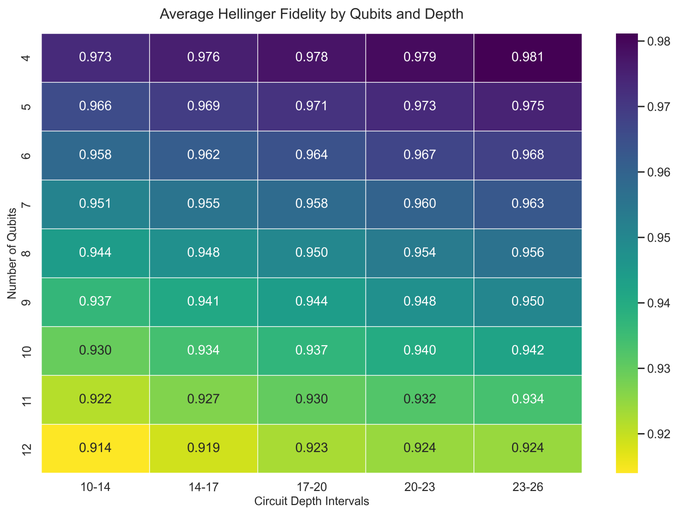

Figure 4. Heatmap of average Hellinger fidelity as a function of qubits and depth.

5. Conclusion

This work quantitatively analyzed the relationship between circuit complexity, statistical sampling, and final fidelity in 100,000 circuits simulated with a physically motivated noise model. The results confirmed our goal of determining how circuit structure and the number of *shots* affect the accuracy of noisy quantum computations. Fidelity decreases with the number of gates and qubits, while circuit depth alone is a weaker predictor.

The analysis also revealed a *fidelity ceiling*, a limit imposed by hardware noise that cannot be overcome by increasing the number of *shots*. These findings suggest that optimizing performance on NISQ devices requires balancing circuit complexity and sampling. Future work will validate these trends on real quantum hardware and explore their use in developing noise-aware compilers and predictive models for estimating circuit fidelity before execution.

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